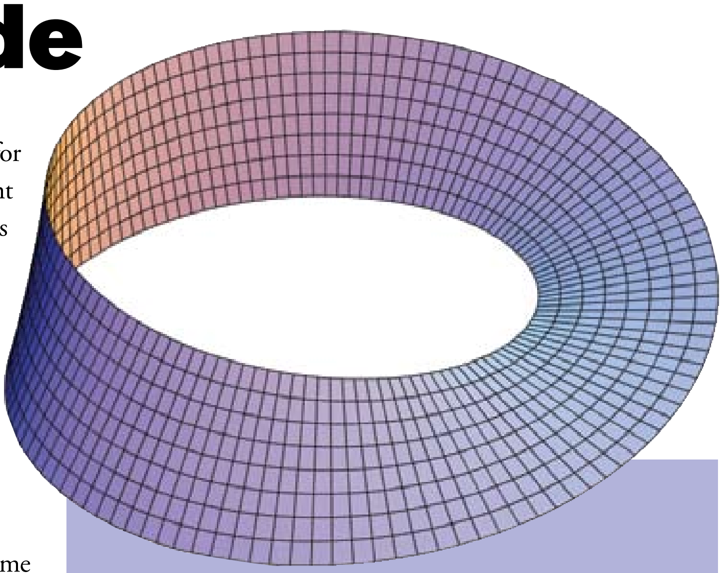
Viewpoint

MODELING

By **GERGELY VASS**

Inside, Outside

When building a virtual model of the real world for computer graphics applications, we usually represent solid objects as polygonal, NURBS, or other types of surfaces. This makes the representation, the modeling, and the rendering of objects much easier. Yet, there are some interesting questions regarding surfaces when representing volumes: Are closed and hole-less surfaces always defining 3D volumes? Do all surfaces have two sides? Why does our favorite modeling application fail at computing the union or the intersection of certain objects? And what is the mysterious error message regarding manifolds?



The Möbius strip is a surface with only one side and only one boundary.

Rigid objects, no matter how thin, always take up some volume in 3D space. It is never a good idea to ignore the thickness of even the finest sheets or plates, especially if they are made of plastic or metal. Our eyes always pick up the highlights on the sharp but still round edges—an essential visual sign of thinness. Modeling polygon meshes that do not perfectly encompass a 3D volume may also cause headaches at different stages of the pipeline: Automatic polygon reduction or collision-detection algorithms likely will fail on our model, we won't be able to use Boolean operations on the object, and rendering artifacts may appear on the final images.

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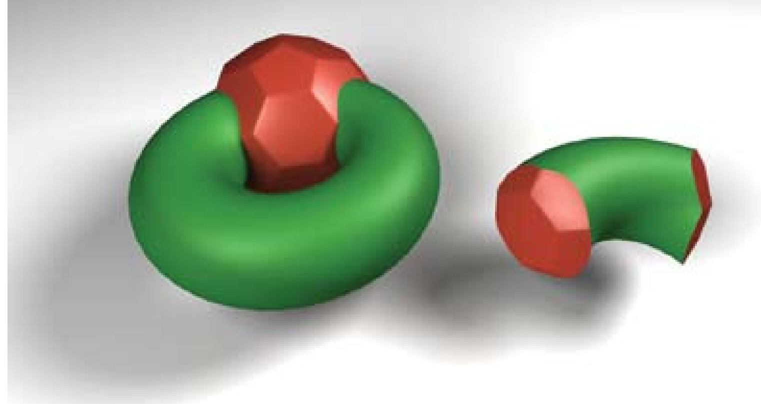
Manifolds

In order to understand the reason why these problems arise, and to be able to fix them, we should look at some results of the theory of geometric topology. The first concept one should understand here is the manifold. Instead of shocking you with some pure mathematical definitions, let's look at the simplest example: the one-dimensional manifold. In a one-manifold, every point has a neighborhood that looks like a line segment. Thus, a one-dimensional manifold is a curved or straight line, with no discontinuities or junctions. Manifolds also can be open (like a curved line) or closed (like a circle).

Two-dimensional manifolds—referring to surfaces in 3D space—have a similar definition: In a two-manifold, every point has a neighborhood that looks like a continuous disk. That is, the surface can be locally deformed into a plane, without tearing it or identifying separate points. An example of a closed, two-dimensional manifold would be a sphere, torus, or the Klein bottle. A valid two-manifold topology polygon mesh never has edges shared by more than two faces (that would be similar to a "T" junction). Essentially, two-manifolds are made of a single sheet of totally flexible, imaginary fabric. Manifolds are defined by a local constraint on the surface. That is good news, as it is very easy to check a polygon mesh by simply inspecting every vertex and edge for problematic topological configurations.

Orientability

Closed, two-manifolds seem to be the perfect abstract representation of boundary surfaces. So, is it fair to say that all meshes with such topology are "solid" models? Well, almost. There is one property, however, that we need to pay close attention to: orientability. In the case of two-manifolds, this property describes whether a



Above illustrates the intersection of two volumes, defined by their boundary surfaces.

surface has two distinct sides. On an orientable surface, it is possible to paint one side blue and the other side green. Most of us would assume that there should be a way to paint the two sides of a surface differently. That is not the case. The Möbius strip—described by German mathematicians August Ferdinand Möbius and Johann Benedict Listing in 1858—is a non-orientable surface and has only one side.

A model of a Möbius strip can be constructed by joining the ends of a paper strip with a single half-twist. If you imagine an ant walking from the seam down the middle of the strip, it will arrive back at the seam, but at the "other side." Walking farther, the ant will reach its starting position again. This single, continuous path demonstrates that the Möbius strip has only one side. In 3D modeling and rendering applications, we usually "mark" the visible side (mostly the outside) of a polygon face by assigning a normal vector to it. While it would be rather easy to model a polygonal Möbius strip in any 3D package, we cannot possibly assign vectors to the faces so that the normals on neighboring polygon faces point in the same direction.

The Möbius strip has had versatile, practical, and symbolical applications, as well. In 1957, the BF Goodrich Company (now Goodrich Corp.) patented a "turnover" conveyor belt system incorporating a half-twisted Möbius band. It had the advantage over conventional systems of a longer life span, as both sides (actually, the only side) were exposed to the same wear and tear. Möbius belts were also used in continuous-loop recording tapes to double the playing time, as well as in typewriter ribbons and computer printer cartridges. The universally recognized recycle logo, developed by 23-year-old USC student Gary Anderson in 1970, is also based on the Möbius strip.

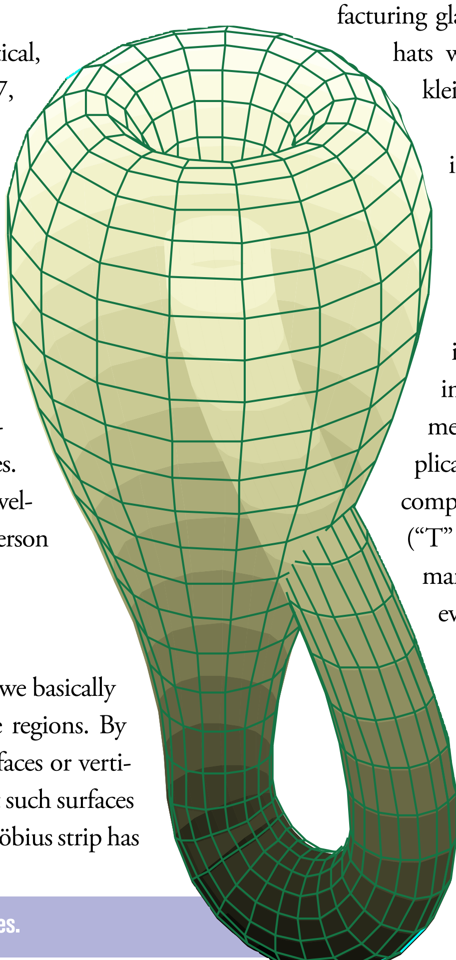
The Klein Bottle

When modeling solids with triangle meshes, we basically divide the 3D space into inside and outside regions. By setting the normal vectors—associated with faces or vertices—to point outside, we rely on the fact that such surfaces have two sides (so they are orientable). The Möbius strip has demonstrated that "open" manifolds are not necessarily orientable. But what about closed surfaces? Are there any closed surfaces with no distinct inner and outer sides? The answer is, once again, yes.

The Klein bottle, first described in 1882 by the German mathematician Felix Klein, is a non-orientable surface with only one side. We can construct a Klein bottle (in a mathematical sense, because it cannot be done without allowing the surface to intersect itself) by joining together the edges of two Möbius strips. Looking at the three-dimensional model of the Klein bottle, we immediately see that the surface is intersecting itself (though in four or higher dimensions, this is not the case). This "minor" technical detail could not stop fans of geometrical topology from manufacturing glass Klein bottles or producing Klein bottle hats with matching Möbius scarves (see www.kleinbottle.com).

In conclusion, polygon meshes of two-manifold topology and with no self-intersection do describe solid objects. The easiest way to create such a model is simple: Start with a valid, basic mesh and use modifications that do not break any of the criteria. Box modeling is a possible approach: We start by defining the desired topology on a (very) low poly mesh. By subdividing and extruding faces, duplicating edge loops, or moving around different components, we will not introduce pathological ("T" or bow-tie shape) faces breaking the two-manifold topology. We should be careful, however, not to introduce self-intersections.

By strictly following these steps, the resulting mesh will have proper normal vector orientation. So, hopefully, we will not see any rendering problems due to reversed faces. Also, we will be able to use Boolean operations, such as difference or union. And other surface operations, like polygon reduction, will also run smoothly on our model. ■



The Klein bottle has no distinct inner or outer sides.